A Possible Explanation of the Missing Deflation Puzzle*

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Abstract

Following the Great Recession, despite large and persistent slowdown in economic activity, the fall in inflation was modest. This is known as the missing deflation puzzle. In this paper, we develop and estimate a New Keynesian model to provide an explanation for the puzzle. The new model allows for time-varying volatility in cross-sectional idiosyncratic uncertainty and accounts for changes in intermediate input prices. We show that inflation did not fall much because intermediate input prices were increasing.

Keywords: Price Mark-up Shocks; Great Recession; Inflation; DSGE; Intermediate Inputs; Input-Output linkages.

JEL Classification Numbers: E31, E32, E52.

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I INTRODUCTION

New Keynesian Dynamic Stochastic General Equilibrium (DSGE) models have become an important tool for monetary policy analysis and forecasting at central banks and other policy institutions around the world. However, the failure of these models to forecast behaviour of inflation and other key macroeconomic variables following the 2008 Great Recession has been interpreted as an evidence against this class of models. Two important papers in this regard are Ball and Mazumder (2011) and Hall (2011). Ball and Mazumder make their point by forecasting inflation for the period between 2008 and 2010 using the New Keynesian Phillips Curve (NKPC), which determines inflation in these models. They find that the NKPC estimated using data from 1960 to 2007 cannot explain inflation over the forecast period. Hall criticises the NKPC on the basis that it fails to provide an explanation for the 'missing deflation' puzzle. Missing deflation is characterised as higher levels of actual inflation than what the NKPC would predict in the aftermath of the Great Recession.

After the collapse of Lehman Brothers in the fourth quarter of 2008, the output gap worsened to more than 6%. While the recession officially ended in the second quarter of 2009, the output gap remained significantly negative for a considerable period after that. It took almost ten years until the output gap was completely closed. As Hall emphasises, given the persistent negative output gap, the NKPC would predict persistent deflation. However, this did not happen. Although inflation fell at the start of the crisis, it soon recovered and remained higher than suggested by the NKPC.

This paper offers an explanation for the missing deflation puzzle. We argue that a reason for the puzzle may be increasing intermediate input prices. Real intermediate input prices experienced a sharp increase between 2009 and 2011 (see Figure 2). While input prices started to fall after 2011, the fall was small and gradual. It took more than four years for prices to return to their 2007 level. Higher intermediate input prices drove up firms' marginal costs, offsetting the deflationary effect of the Great Recession. Therefore, inflation did not fall as much as it otherwise would have during the post-2008 period.

We test our argument by using a modified version of the Smets and Wouters (2007) (henceforth SW) model. Specifically, we reformulate the SW model to include the financial frictions mechanism in Bernanke et al. (1999) (henceforth BGG) and to account for changes in intermediate input prices. Further, we remove the price mark-up shocks in the model and, following Aoki (2001), De Walque et al. (2006) and Huang and Liu (2005), consider supply-side shocks that arise from changes in relative intermediate input prices. Let us briefly explain these additions to the SW model.

To incorporate intermediate prices in the SW model, we divide production into two sectors. In one of the two sectors intermediate inputs are produced. The second sector produces finished goods. Marginal costs in the finished goods sector also depend on the relative price of intermediate inputs. Prices in both sectors are set according to Calvo (1983) pricing. Therefore, inflation in both sectors depend on sector-specific current and future marginal costs. We further assume that prices in the intermediate inputs sector are subject to a sector-specific shock.[1] As a result, in addition to marginal costs, inflation in the intermediate inputs sector also depends on the sector-specific input-price shock.

Turning to the second addition, as is well-known (see, e.g. Christiano et al. (2014), henceforth CMR), the BGG mechanism models the idiosyncratic uncertainty faced by entrepreneurs. The common assumption is that the volatility of cross-sectional idiosyncratic uncertainty fluctuates over time. This measure of volatility is referred to as risk. In line with CMR, we assume that the risk shock process has both unanticipated (or stochastic) and anticipated (or news) components. Several recent papers (e.g. CMR and Schmitt-Grohe and Uribe (2012)) show that accounting for the anticipated component improves the empirical performance of the model significantly. The rest of the model is exactly the same as that in SW.

Next, we estimate the new model for US data using Bayesian techniques for the period from 1965Q1 until 2013Q2. We then back out input-price shocks to the intermediate inputs producing sector. Finally, we compare the dynamics of inflation and marginal costs from our model, with and without input-price shocks, for the period when the output gap remained significantly negative: from 2009 until 2013.

The results confirm our suggestion that intermediate input prices played a crucial

[1]Here we do not seek to provide an answer to another important question: What drives intermediate input prices? It is possible that most intermediate input prices are determined in international auction markets. Feenstra and Weinstein (2017) show that globalisation has increased the share of foreign firms in US domestic absorption. Fally (2012) shows that value addition has shifted from upstream industries towards downstream industries. A shift in value addition away from intermediate industries and an increase in competition from foreign firms may have decreased market power in upstream industries. This could explain the increase in the degree of synchronisation between intermediate input prices and global economic activity. We leave exploring this further for future research. The sector-specific shock in the intermediate sector is meant to capture such factors affecting intermediate input prices.

role in explaining inflation dynamics during the period when the output gap was significantly negative. When we shutdown shocks to intermediate input prices, the counterfactual inflation and marginal costs fall persistently following the crisis - but not, when feeding in the backed out input-price shocks. This is because, in our model, inflation depends not only on the output gap, as in the standard New Keynesian model, but also on the relative price of intermediate inputs. Therefore, the increase in intermediate input prices offsets most of the decrease in marginal costs driven by the persistent slowdown in economic activity after the collapse of Lehman Brothers.

Turning to the role of financial frictions, the BGG mechanism plays a crucial role in explaining output dynamics in the model. It helps capture the drop in output at the beginning of the crisis. We find that both components of the risk shock process, anticipated and unanticipated, are important for capturing the fall in output. The intuitive explanation for the importance of the anticipated component is as follows. Anticipating that future uncertainty will increase, banks increase the interest rate they charge on loans. An increase in the interest rate further depresses investment, thus leading to a larger fall in output and, consequently, inflation.[2] However, the fall in inflation is offset by the increase in intermediate input prices.

This paper is closely related to earlier papers by Coibion and Gorodnichenko (2015) and Del Negro et al. (2015) (henceforth NGS). Coibion and Gorodnichenko (2015) show that 'missing deflation' is a one-off event in response to rising oil prices. However, in our model, accounting for oil prices alone does not have a significant

[2]In the previous version of this paper, we also find the anticipated component of the risk shock to play an important role in correctly forecasting observed contraction in output growth at the start of the crisis.

effect on inflation. This is because, at around 1%, the share of oil in production is very small. Further, our paper differs from Coibion and Gorodnichenko in its modelling approach. While their analysis is based on the expectations−augmented Phillips curve proposed by Friedman (1968), ours is carried out in a New Keynesian general equilibrium framework in which the Phillips curve is micro-founded. Nevertheless, by focusing on intermediate input prices instead of oil prices, this paper further strengthens their conclusion by showing that their finding of missing deflation being a one-off event has a wider applicability and holds also in a New Keynesian general equilibrium model.

NGS, on the other hand, employ a New Keynesian model with BGG-type financial frictions and argue that the near stability of inflation following the Great Recession was due to anchored expectations. Their results depend on having a large degree of price stickiness and, therefore, a very flat NKPC. At 8 quarters, average age of price contracts in NGS is twice that in micro evidence on prices (Klenow and Malin (2011)). NGS suggest that since inflation expectations remained anchored, prices were not revised downwards substantially despite sharp contraction in output.

Another possible explanation for the stability of inflation in aftermath of the Great Recession is forwarded by Gilchrist et al. (2016). Gilchrist et al. note that financially constrained firms raise their prices following adverse financial shocks. They do this because financially constrained firms find it difficult to access external finance, since they face a higher risk of default. As a result, they raise their prices to maintain internal liquidity even at the cost of a decrease in firm's market share. On the other hand, firms that are not financially constrained cut their prices in response to a decrease in demand for their products. The explanation in Gilchrist et al. and the one provided in this paper have important implications for firms' price mark-ups: In Gilchrist et al., mark-ups increase because financially constrained firms raise their prices in order to maintain internal liquidity. Whereas the explanation in this paper implies increasing mark-ups for intermediate inputs producing firms and decreasing mark-ups for finished goods producing firms.

The implication for finished goods firms' mark-ups in this paper is in line with the explanation for missing deflation suggested in Christiano et al. (2015). Christiano et al. propose that inflation did not fall due to increases in firms' marginal costs. However, the reason for increasing marginal costs is different in Christiano et al. than in this paper. Following the Great Recession, borrowing costs increased substantially. Therefore, financially constrained firms that were previously financing their operating costs (e.g. wage bills) through borrowing experienced an increase in their financing costs. This increased firms' marginal costs and, therefore, kept inflation stable.

The rest of this paper is organised as follows. Section II describes the model. Section III and IV explain the estimation strategy and present estimation results, respectively. Section V analyses dynamics of inflation with and without input-price shocks. Section VI explores how expectations about long-run inflation interact with input-price shocks in our model. We also test if our results are sensitive to alternative values of intermediate share in finished goods production. Finally, Section VII concludes.

II THE MODEL

The model in this paper builds on the model by SW to allow for input-output linkages between intermediate inputs and finished goods producing firms. It also accounts for the idiosyncratic uncertainty faced by entrepreneurs. While production of intermediate inputs requires labour and capital as only factors of production, production of finished goods also requires intermediate inputs as an additional factor input. The two sectors also face the financial accelerator mechanism of BGG where financial market frictions arising through information asymmetry and agency costs affect the real side of the economy. In this we follow the work of NGS and CMR. Finally, the modelling of households and the monetary policy is standard New Keynesian.

In the rest of this section, we describe the behaviour of firms in the two sectors followed by description of the BGG financial frictions. The behaviour of households and monetary authority is similar to SW and is, therefore, not included here for brevity. The model is detrended using a deterministic labour-augmenting trend. For estimations, the model is linearised around the stationary steady-state of detrended variables. All nominal variables are expressed in capital letters. Variables which are written in small letters in log-linearised equations are real expressions of their nominal counterparts and are in terms of deviation from their corresponding steady-state value.

II.I Intermediate and Finished Goods

There is a continuum of firms in each sector: a finished goods sector (s); and, an intermediate inputs sector (m). Firms in both sectors produce under an imperfectly competitive market and have monopoly power over a differentiated good. Each firm within two sectors produces a single differentiated good, $Y^s_{f,t}$ and $Y^m_{j,t}$, respectively. These sector-specific differentiated goods are then combined to produce a sector-specific final good according to:

$$Y^s_t = \left[\int_0^1 (Y^s_{f,t})^{\frac{\rho-1}{\rho}} df\right]^{\frac{\rho}{\rho-1}} \tag{1}$$

$$Y^m_t = \left[\int_0^1 (Y^m_{j,t})^{\frac{\rho-1}{\rho}} dj\right]^{\frac{\rho}{\rho-1}} \tag{2}$$

where ρ is elasticity of substitution between sector-specific differentiated goods.

Firms in the finished goods sector use labour, capital and intermediate inputs as factors of production. The production function of firms is given by:

$$Y^s_{f,t} = \left[Y^m_{f,t}\right]^{\mu} \left\{A_t {K^s_{f,t}}^{\alpha} \left[\gamma^t L^s_{f,t}\right]^{1-\alpha}\right\}^{1-\mu} - \gamma^t \Phi \tag{3}$$

where $Y^m_{f,t}$ is intermediate sector goods used as an additional input by firm f in the finished goods sector. $L^s_{f,t}$ is a composite of labour input and $K^s_{f,t}$ is capital services. μ and Φ is input-output elasticity and fixed costs, respectively. γ^t represents the labour-augmenting deterministic growth rate in the economy. A_t is the productivity shock which follows an AR(1) process of the form:

$$a_t = \rho_a a_{t-1} + \epsilon_{a,t} \tag{4}$$

where $a_t = lnA_t$ and ρ_a determines persistence of the productivity shock process. $\epsilon_{a,t}$ is the i.i.d. shock with mean zero and standard deviation σ_a.

Firms minimise their costs in equation (5) subject to their production function:

$$\min W_t L^s_{f,t} + R_{k,t} K^s_{f,t} + P_{m,t} Y^m_{f,t} \tag{5}$$

where W_t, $R_{k,t}$ and $P_{m,t}$ is the nominal wage rate, rental rate of capital and intermediate input price, respectively. Cost minimisation problem gives the following log-linearised expression for marginal costs:

$$mc_t^s = (1-\mu)(\alpha r_t^k + (1-\alpha)w_t) + \mu p_t^m - a_t \tag{6}$$

where mc_t^s denotes real marginal costs in the finished goods sector. The demand function for intermediate inputs is given by:

$$y_{f,t}^m = mc_t^s - p_t^m + y_{f,t}^s \tag{7}$$

Firms in the finished goods sector take input prices as given. Finished goods prices are set according to Calvo (1983) with no ad-hoc price indexation. The log-linearised NKPC in this sector is given by:

$$\pi_t^s = \beta\gamma^{1-\sigma_c}\pi_{t+1}^s + \kappa^s mc_t^s \tag{8}$$

where π_t^s is inflation in the finished goods sector. κ^s is given by

$$\kappa^s = \frac{(1-\zeta_p\beta\gamma^{1-\sigma_c})(1-\zeta_p)}{\zeta_p} \tag{9}$$

where ζ_p is the Calvo parameter for price stickiness and β is the discount factor. σ_c represents the elasticity of intertemporal substitution.

Unlike in the finished goods sector, firms in the intermediate inputs sector only use labour and capital as two factors of production to produce a differentiated good. The production function in this sector is given by:

$$Y_{j,t}^m = A_t\big[K_{j,t}^m\big]^{\alpha}\big[\gamma^t L_{j,t}^m\big]^{1-\alpha} - E_t\gamma^t\Phi \tag{10}$$

where $L_{j,t}^m$ is a composite of labour input and $K_{j,t}^m$ is capital services used in the intermediate sector by firm j. α and Φ are capital-output elasticity and fixed costs, respectively.

The production function in equation (10) also includes a shock to fixed costs (E_t). This shock is meant to capture changes in production that arise from external factors, such as unusually cold winters and rare disasters.[3] E_t follows an ARMA(1,1) process of the form:

$$e_t = \rho_e e_{t-1} + \epsilon_{e,t} - \mu_e \epsilon_{e,t-1} \tag{11}$$

where $e_t = lnE_t$ and ρ_e determines persistence of the shock process. $\epsilon_{e,t}$ is the i.i.d. shock with mean zero and standard deviation σ_e. We assume that the shock affects the intermediate inputs sector only. However, it has an indirect effect on the finished goods sector. An unusually cold winter would cause a disruption in the production of intermediate inputs. A decrease in the supply of intermediate inputs will consequently affect finished goods production as well.[4]

Firm j in the intermediate inputs sector solves the following cost minimisation problem:

$$\min W_t L^m_{j,t} + R_{k,t} K^m_{j,t} \tag{12}$$

The log-linearised expression for real marginal costs is given by:

$$mc^m_t = \alpha r^k_t + (1-\alpha) w_t - a_t \tag{13}$$

As in the finished goods sector, firms in the intermediate inputs sector also set their prices according to Calvo (1983) with no ad-hoc price indexation. The NKPC in the intermediate inputs sector is given by:

$$\pi^m_t = \beta \gamma^{1-\sigma_c} \pi^m_{t+1} + \kappa^m (mc^m_t - p^m_t) + \varphi_t \tag{14}$$

[3] Another reason for including the E_t shock is a technical one. Since we include an additional data series on intermediate prices in our estimations, we need an additional shock to ensure that the number of observed variables are equal to the number of shocks. In any case, E_t does not play a significant role in driving model results. A variance decomposition analysis suggests that this shock explains only about 3.35% and 1.81% of fluctuations in output growth and inflation, respectively. In an alternative setting, following Barro (2006) and Gourio (2012), we model the E_t shock as a shock to capital. Our main conclusions are robust to this alternate specification.

[4] In an alternative setting, we assume that the shock affects both the intermediate inputs and the finished goods sectors directly. Doing so does not change our main results significantly.

where π_t^m is inflation in the intermediate inputs sector. κ^m is the slope coefficient of the form:

$$\kappa^m = \frac{(1-\zeta_p^m\beta\gamma^{1-\sigma_c})(1-\zeta_p^m)}{\zeta_p^m} \tag{15}$$

where ζ_p^m is the Calvo parameter for price stickiness specific to the intermediate sector. φ_t in equation (14) is an input-price shock which is intended to capture international factors driving intermediate input prices. φ_t follows an ARMA(1,1) process of the form[5]:

$$\varphi_t = \rho_\varphi \varphi_{t-1} + \epsilon_{\varphi,t} - \mu_\varphi \epsilon_{\varphi,t-1} \tag{16}$$

The following subsection describes the financial accelerator mechanism which is identical to that in NGS.

II.II The Financial Accelerator Mechanism and the Risk Shock

The introduction of financial frictions in the model alters the arbitrage equation. The arbitrage equation between the return on capital and the riskless rate in SW is replaced with an equation for capital returns and an equation for the spread between capital returns and the riskless rate. The equation determining the spread is:

$$E_t[\tilde{R}_{t+1}^k - R_t] = b_t + \zeta_{sp,b}(q_t^k + \bar{k}_t - n_t) + \tilde{\sigma}_{w,t} \tag{17}$$

Equation (17) has the SW arbitrage equation as a special case when the parameter ($\zeta_{sp,b}$) associated with the ratio of the value of installed capital to net worth ($\frac{Q_{t+i-1}^k \bar{K}_{t+i-1}}{N_{t+i-1}}$) is zero. q_t^k is the real value of the capital stock, $\bar{k}_t$ is capital stock and n_t is net worth of entrepreneurs. $\tilde{\sigma}_{w,t}$ is the risk shock and $\tilde{R}_t^k$ denotes capital return

[5] Our results are robust to alternate specifications for the shock process.

to the entrepreneurs. $\tilde{R}^k_t$ can also be interpreted as required returns on capital, since entrepreneurs' borrowing costs within the model always equal $\tilde{R}^k_t$, and is given by:

$$\tilde{R}^k_t - \pi_t = \frac{r^k_*}{r^k_* + (1-\delta)} r^k_t + \frac{1-\delta}{r^k_* + (1-\delta)} q^k_t - q^k_{t-1} \tag{18}$$

where r^k_* is the steady-state rental rate of capital and δ is the rate of depreciation of capital stock. Entrepreneurs' net worth (n_t) evolves according to:

$$n_t = \zeta_{n,\tilde{R}^k}(\tilde{R}^k_t - \pi_t) - \zeta_{n,R}(R_{t-1} - \pi_t) + \zeta_{n,q^k}(q^k_{t-1} + \bar{k}_{t-1}) + \zeta_{n,n} n_{t-1} \tag{19}$$

Following CMR and Fernandez-Villaverde et al. (2011), we assume the following process for the risk shock:

$$\tilde{\sigma}_{w,t} = \rho_{\tilde{\sigma}} \tilde{\sigma}_{w,t-1} + u_{\tilde{\sigma},t} \tag{20}$$

where

$$u_{\tilde{\sigma},t} = \rho_{\tilde{\sigma},n} u_{\tilde{\sigma},t-1} + \epsilon_{\tilde{\sigma},t} \tag{21}$$

After straightforward algebra, the last two equations can be rewritten as:

$$\tilde{\sigma}_{\omega,t+i} = \rho_{\tilde{\sigma}} \tilde{\sigma}_{\omega,t+i-1} + \rho^i_{\tilde{\sigma},n} \boldsymbol{\epsilon_{\tilde{\sigma},t}} + \rho^i_{\tilde{\sigma},n} \sum_{j=1}^{\infty} \rho^j_{\tilde{\sigma},n} \epsilon_{\tilde{\sigma},t-j} \tag{22}$$

where $0 < \rho_{\tilde{\sigma}}, \rho_{\tilde{\sigma},n} < 1$ and $\epsilon_{\tilde{\sigma},t}$ is i.i.d. (independent and identically distributed) and denotes the unanticipated component of risk $(\tilde{\sigma}_{\omega,t})$. Eq. (22) is an attempt to mimic the effect of the Lehman shock which increased both current and future risk in the economy. To see this more clearly, consider a financial shock $(\boldsymbol{\epsilon_{\tilde{\sigma},t}})$ in period 't'. $\boldsymbol{\epsilon_{\tilde{\sigma},t}}$ affects the economy in period 't' via two channels. First, $\boldsymbol{\epsilon_{\tilde{\sigma},t}}$ increases risk in period 't' $(\tilde{\sigma}_{\omega,t})$. Second, it also increases future risk $(\tilde{\sigma}_{\omega,t+i})$ and thus affects the current state of the economy through agents' intertemporal adjustment. $\boldsymbol{\epsilon_{\tilde{\sigma},t}}$ will receive less weight the further agents look into the future. $\rho^i_{\omega,n}$ is the weight on $\epsilon_{\tilde{\sigma},t}$ for risk in period '$t + i'$.

We call $\epsilon_{\tilde{\sigma},t-j}$ an anticipated component whose value was revealed in $t-j$. Thus, at time t the realisation of the risk $\tilde{\sigma}_{\omega,t}$ is influenced by the combined impact of both the unanticipated and the anticipated components. Furthermore, as Christiano et al. (2010) argue, such a generalised shock process helps to "tackle the deep-seated misspecification problems in DSGE models."

The rest of model equations are the same as in SW and are listed in the Online Appendix.

III ESTIMATION STRATEGY

We estimate our model for the US economy for the period from 1965Q1 to 2013Q2 using Bayesian estimation techniques.[6] In the estimation, we use ten macroeconomic series at the quarterly frequency. Six of these series are the same as those employed by SW. These series are the log difference of real GDP, real consumption, real investment, real wage, log hours worked and log difference of the GDP deflator.

We use the shadow federal funds rate, as estimated in Wu and Xia (2016), in estimations, instead of the federal funds rate.[7] The reason why we use the shadow rate is that our estimation period includes a period of zero lower bound and unconventional monetary policies. As a result, the federal funds rate may not capture the actual stance of monetary policy during this period. Unlike the federal funds rate, the shadow rate is not constrained by the zero lower bound. Wu and Xia show

[6] We ensure an acceptance rate of around 30% and allow for 250,000 replications for the Metropolis-Hastings algorithm. Estimation is done in Dynare 4.5.3.

[7] A detailed discussion on how the shadow rate is estimated is available in Wu and Xia (2016). The series for the shadow rate is available on the Atlanta Fed's website.

that the shadow funds rate better captures the stance of monetary policy during this period.

The additional series we employ are data on credit spreads, 10-year inflation expectations and the log difference of real intermediate input prices. The credit spread is the difference between the interest rate on BAA-rated corporate bonds and the 10 year US government bond rate. We obtain data for 10-year inflation expectations from Blue Chip Economic Indicators and Professional Forecasters surveys. Using data on inflation expectations is helpful since, as pointed out by Del Negro and Eusepi (2011) and Kiley (2008), inflation expectations contain information about people's beliefs regarding the FED's inflation objectives. Survey data indicate that long-run inflation expectations remained anchored to the FED's inflation target throughout the relevant period. Using inflation expectations data can capture this fact. We discuss the importance of inflation expectations for our results in more detail in section VI.

The intermediate input price data we use to identify input-price shocks are from the St. Louis FED database. The specific series we employ is Producer Price Index by Commodity for Intermediate Demand by Commodity Type: Processed Goods for Intermediate Demand (WPSID61).[8]

[8] This series is part of the BLS forward-flow model of production and price change (i.e. FD-ID system). This model organises commodities (i.e. goods and services) into stages and tracks input price changes at each stage, using BEA's commodity-consumption and industry-production data. Commodities included in the preceding stage are primarily used as inputs in production of commodities in the current stage (See page 11 and chapter 14 of BLS Handbook of Methods for further details, which is available here: https://www.bls.gov/opub/mlr/2011/02/art1full.pdf). This approach is useful since it allows for studying how price shocks are transmitted forward from preceding to subsequent stages of production. The intermediate price index chosen in this paper is the index representing input prices for finished goods producing firms (i.e. the final production stage).

Measurement equations relating data to model variables are:

$$
\begin{aligned}
&OutputGrowth &&= \gamma + 100(y_t - y_{t-1}) \\
&ConsumptionGrowth &&= \gamma + 100(c_t - c_{t-1}) \\
&InvestmentGrowth &&= \gamma + 100(i_t - i_{t-1}) \\
&RealWageGrowth &&= \gamma + 100(w_t - w_{t-1}) \\
&HoursWorked &&= \bar{l} + 100 l_t \\
&Inflation &&= \pi_* + 100\pi_t \\
&ShadowFederalFundsRate &&= R_* + 100 R_t \\
&Spread &&= SP_* + E_t[\tilde{R}^k_{t+1} - R_t] \\
&10yrInflExp &&= \pi_* + E_t[\frac{1}{40}\Sigma_{k=1}^{40}\pi_{t+k}] \\
&IntermediateInflation &&= \pi_\varphi + 100(p^m_t - p^m_{t-1})
\end{aligned}
\tag{23}
$$

where $\bar{l}$, $\pi_* = 100(\Pi_* - 1)$ and $R_* = 100(\beta^{-1}\gamma^{\sigma_c}\Pi_* - 1)$ are the steady-state of the quarterly hours worked, inflation and nominal interest rates, respectively. π_φ and SP_* are the steady-state of intermediate price inflation and credit spread, respectively. All variables are expressed in percent.

Table 3 and Table 4 summarise our assumptions regarding prior distributions. Priors for most of model parameters are similar to those in SW. Calvo parameters for intermediate and finished goods sectors are specified a Beta prior distribution with standard deviation of 0.10. Surveying the literature starting from 1920s, Goldberg and Hellerstein (2011) note that "the conventional wisdom in the literature has come to be that producer prices are more rigid than ... consumer prices". However, using micro-data on prices complied by the US BLS for the period from 1987 to 2008,

Table 1: **Exogenous parameter values**

Parameter	Definition	Values
β	Discount factor	0.9995
γ	Trend growth rate	1.004
δ	Depreciation rate	0.025
ϵ_w	Curvature of the Kimball labour market aggregator	10
g_y	Government spending-output ratio	0.18
μ	Share of intermediate inputs in finished goods firms' production	0.60
μ^u	Share of sector-specific labour and capital in aggregate labour and capital	0.50

these authors then go on to find that producer prices for finished goods have more or less the same price rigidity as consumers prices. Focussing on the period from 1998 to 2005, Nakamura and Steinsson (2008) also reach similar conclusion. Nakamura and Steinsson (2008) further report that intermediate input prices are more flexible than both finished goods producer prices and consumer prices. Despite the fact that Nakamura and Steinsson focus on a more recent period (our dataset starts from 1965), we assume that the prior mean for the Calvo parameter is lower for input prices than for finished goods prices. Specifically, the prior mean for the Calvo parameter in the intermediate inputs sector is assumed to be 0.40, while the corresponding parameter in the finished goods sector is set at 0.75.

Following Huang and Liu (2005), the share of intermediate inputs in finished goods production (μ) is calibrated to 60%. We assume that aggregation is done using a Dixit and Stiglitz aggregator and therefore ϵ_p equals 1. Table 1 reports values for parameters that are fixed in estimation.

We now turn to parameter values for the financial sector. Following CMR, we

Table 2: **Financial Frictions: Exogenous parameter values**

Entrepreneurs:		
$F^*(\bar{\omega})$	Percent of businesses that go into bankruptcy in a year	0.01
$Var(log\omega)$	Variance of the log-normally distributed i.i.d shock	0.24
τ	Fraction of entrepreneurs surviving to the next period	0.9728
μ^e	Monitoring costs	0.31
r_*^k	Rental rate of capital	0.045

calibrate the survival rate of entrepreneurs (τ) as 97.28% and the percentage of businesses going bankrupt ($F^*(\bar{\omega})$) as 1% annually. To match the risk premium in the steady-state, the rental rate of capital is assumed to be 0.045. $Var(log\omega)$ is set at 0.24. Different from CMR, μ^e is endogenous in our model and has a steady-state value of 0.31, which is less than the value of 0.94 assumed in CMR. Parameters in the net worth equation are also endogenous. All these numbers are summarised in Table 2.

We estimate two financial sector parameters in Equations (17) and (23), $\zeta_{sp,b}$ and SP_*, respectively. Priors for financial sector parameters are set in line with NGS and are given in Table 4. SP_* follows a Gamma distribution with prior mean of 2 and standard deviation of 0.10. $\zeta_{sp,b}$ is assumed to follow a Beta distribution with mean of 0.05 and standard deviation of 0.005. Three parameters related to the risk shock are the persistence of the shock process ($\rho_{\bar{\sigma}}$), the standard deviation of the shock ($\sigma_{\bar{\sigma}}$) and the parameter on the anticipated components of the risk shock ($\rho_{\bar{\sigma},n}$). $\rho_{\bar{\sigma}}$ has a Beta prior distribution with mean 0.75 and standard deviation 0.15. $\sigma_{\bar{\sigma}}$ follows an Inverse Gamma distribution with mean 0.05 and standard deviation 4. $\rho_{\bar{\sigma},n}$ also follows an Inverse Gamma prior distribution with mean 1 and standard deviation 2.

The risk shock follows a process that allows for anticipated signals as explained in equation (20). The price mark-up shock in SW is replaced with the two supply side shocks, φ_t and e_t. We interpret φ_t in equation (14) as shocks arising from changes in real intermediate input prices. Persistence parameters of the two shock processes follow a beta prior distribution with mean 0.50 and standard deviation 0.20. The standard deviation of the intermediate input shock (σ_φ) has an Inverse Gamma prior distribution with mean 1 and standard deviation 2. σ_e also follows an Inverse Gamma distribution with mean 1 and standard deviation 2. Prior distributions of remaining parameters in the model are identical to those in SW.

IV ESTIMATION RESULTS

Estimated values for structural parameters are reported in Table 3. Table 3 also includes prior and posterior standard deviations for corresponding parameters.

The posterior mean of the price stickiness parameter (ξ_p) in the finished goods sector is 0.74, suggesting an average age of price contract of about 4 quarters. In contrast, when estimated over the sample period including Great Recession, NGS and SW models suggest an average age of price contract of around 8 quarters. The estimated value of ξ_p^m is 0.92, suggesting that intermediate input prices are stickier than finished goods prices. This finding is consistent with the "conventional wisdom" emphasised by Goldberg and Hellerstein (2011). When we estimate the model using a shorter sample starting from 1981, results are consistent with the findings in Nakamura and Steinsson (2008) that intermediate input prices are more flexible than finished goods prices. Regardless of which sample we use, our conclusions remain

Table 3: **Prior and Posterior Estimates of Structural Parameters**

	Prior Distribution			Posterior Distribution	
	type	Mean	st. dev.	Mean	st. dev
structural parameters:					
φ	Normal	4.000	1.500	5.766	0.566
σ_c	Normal	1.500	0.375	1.272	0.045
h	Beta	0.700	0.100	0.553	0.034
ξ_w	Beta	0.500	0.100	0.927	0.003
ι_w	Beta	0.500	0.150	0.429	0.059
σ_l	Normal	2.000	0.750	2.084	0.216
ξ_p^s	Beta	0.750	0.100	0.739	0.009
ξ_p^m	Beta	0.400	0.100	0.921	0.025
ψ	Beta	0.500	0.150	0.475	0.043
ϕ_p	Normal	1.250	0.125	1.386	0.045
r_π	Normal	1.500	0.250	1.471	0.038
ρ_r	Beta	0.500	0.100	0.729	0.018
r_y	Normal	0.750	0.050	0.629	0.009
π_s^*	Gamma	0.625	0.100	0.628	0.034
π_m^*	Normal	0.000	1.000	-0.325	0.039
$\underline{\beta}$	Gamma	0.250	0.100	0.079	0.025
$\bar{l}$	Normal	0.000	2.000	1.504	0.472
γ	Normal	0.400	0.100	0.381	0.013
α	Normal	0.300	0.050	0.178	0.009
SP_*	Beta	2.000	0.100	1.838	0.028
$\zeta_{sp,b}$	Beta	0.050	0.005	0.041	0.001

unchanged.

Posterior estimate for ξ_w further suggests that wages are more sticky than finished goods prices. The estimate of α is 0.18 and is similar to that reported in SW. Posterior estimates of parameters governing monetary policy are consistent with empirical evidence: r_ϕ and r_y have a posterior mean of 1.47 and 0.63, respectively. The persistence parameter of the input-price shock (ρ_φ) is estimated at 0.30. Reflecting the highly volatile nature of intermediate input prices, the standard deviation of input-price shocks is large at 1.39.

Table 4: **Prior and Posterior Estimates of Shock Processes**

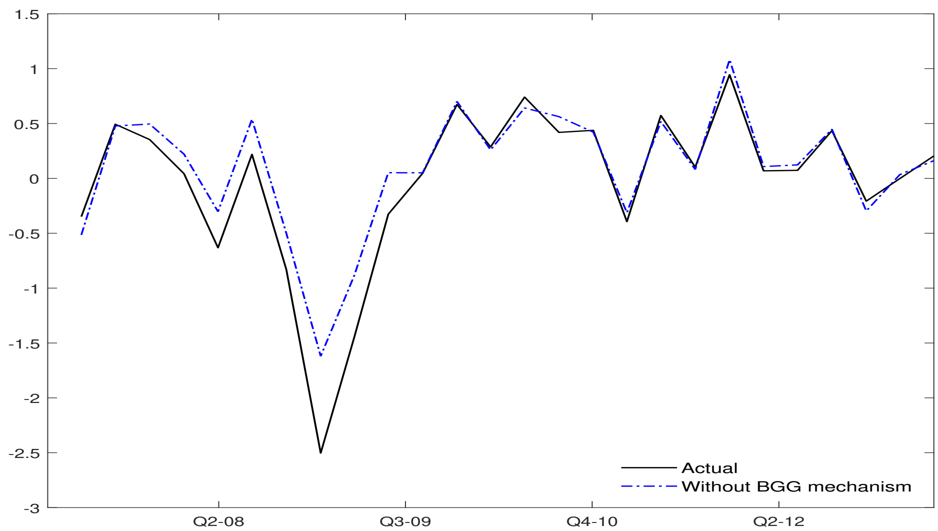
	Prior Distribution			Posterior Distribution	
	type	Mean	st. dev.	Mean	st. dev
persistence of exogenous shocks:					
ρ_a	Beta	0.500	0.200	0.940	0.006
ρ_φ	Beta	0.500	0.200	0.300	0.040
ρ_{ei}	Beta	0.500	0.200	0.973	0.007
$\rho_{\bar{\sigma}}$	Beta	0.750	0.150	0.457	0.062
$\rho_{\bar{\sigma},n}$	Beta	0.750	0.150	0.995	0.003
ρ_b	Beta	0.500	0.200	0.988	0.002
ρ_g	Beta	0.500	0.200	0.989	0.004
ρ_μ	Beta	0.500	0.200	0.996	0.001
ρ_r	Beta	0.500	0.200	0.109	0.036
ρ_w	Beta	0.500	0.200	0.510	0.043
ρ_{π^*}	Beta	0.500	0.200	0.891	0.012
μ_φ	Beta	0.500	0.200	0.148	0.066
μ_{ei}	Beta	0.500	0.200	0.275	0.057
μ_w	Beta	0.500	0.200	0.515	0.039
ρ_{ga}	beta	0.500	0.200	0.027	0.012
σ_a	Inv.Gamma	0.100	2.000	1.409	0.106
σ_φ	Inv.Gamma	1.000	2.000	1.389	0.207
σ_{ei}	Inv.Gamma	1.000	2.000	4.347	0.363
$\sigma_{\bar{\sigma}}$	Inv.Gamma	0.050	4.000	0.073	0.005
σ_b	Inv.Gamma	0.100	2.000	0.012	0.001
σ_g	Inv.Gamma	0.100	2.000	0.514	0.028
σ_μ	Inv.Gamma	0.100	2.000	0.298	0.029
σ_r	Inv.Gamma	0.100	2.000	0.269	0.017
σ_w	Inv.Gamma	0.100	2.000	0.346	0.023
σ_{π^*}	Inv.Gamma	0.100	2.000	0.129	0.017

V Results

Our model is built on the idea that to capture post-2008 inflation dynamics, it is essential to account for shocks to intermediate input prices. In this section, we test the validity of this idea using our estimated model. Specifically, we study the role of such shocks for inflation after the Great Recession. We also discuss the dynamics of output, because the relationship between inflation and output is a defining feature of the NKPC. We start by discussing output dynamics.

As noted by NSG and as is shown in Figure 1, incorporating BGG-type financial

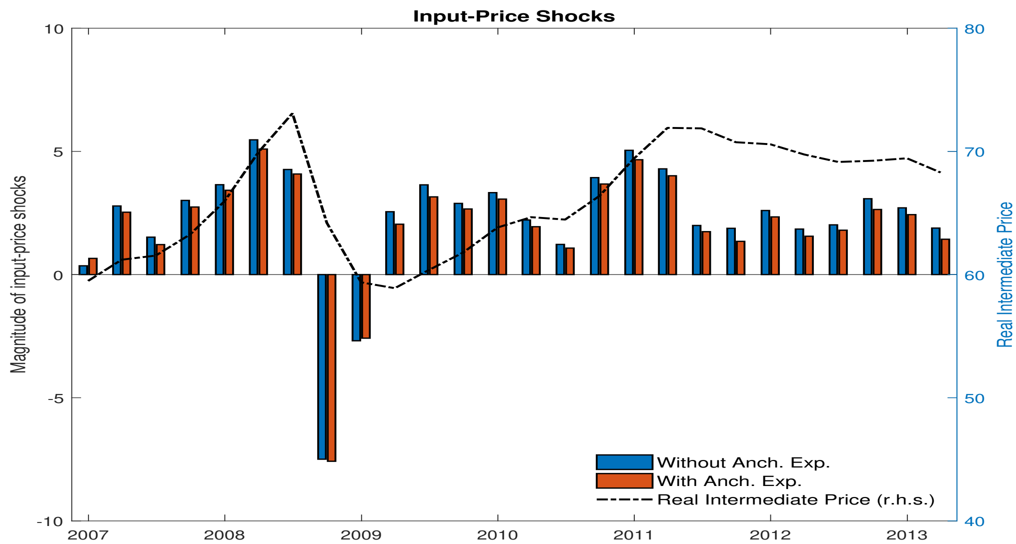
Figure 1: **Output Growth Simulations**



Note: *The solid black line is observed output growth. Dotted-dashed blue line is the simulation with no BGG mechanism.*

frictions is crucial for the model to capture output dynamics during the post-2008 period. Figure 1 plots output growth when the model is simulated without the BGG mechanism. We obtain this simulated series by setting financial frictions shocks to zero. Figure 1 also plots the corresponding series from the benchmark model with the BGG mechanism. The BGG mechanism plays an important role in explaining output dynamics of the Great Recession between 2008 and 2009. The fall in output growth is one percentage point less in absence of adverse financial shocks than the observed fall. While such shocks affect inflation through their effect on output, a shock decomposition analysis (see Figure 3) suggests that the direct contribution of

Figure 2: **Input-Price Shocks**



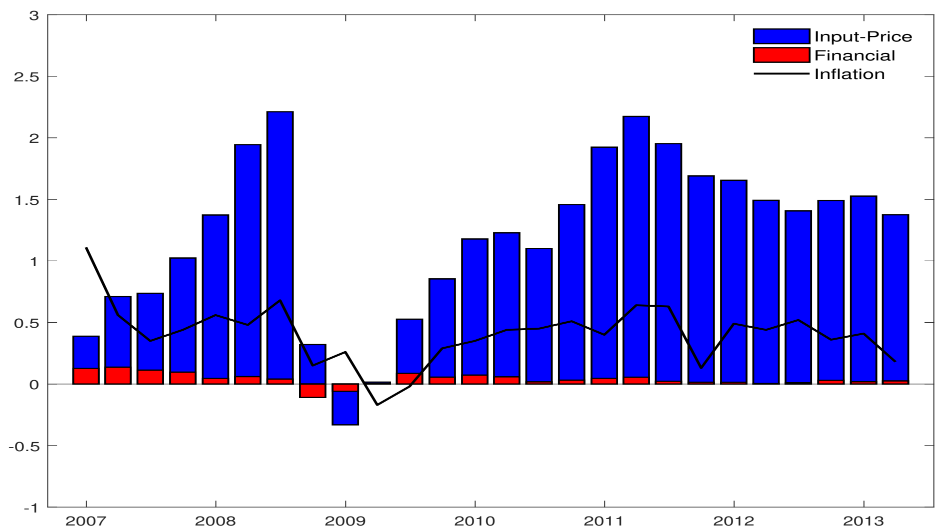
Note: *This figure plots input-price shocks from baseline estimations (orange) and also from a counterfactual exercise where inflation expectations are not anchored (blue). The dash-black line plots real intermediate input prices. The figure is plotted for the period from 2007Q1 until 2013Q2.*

such shocks for inflation is small.

We now turn to studying the role of intermediate input prices in explaining inflation during the post-2008 period (from 2009Q1 to 2013Q2). To achieve this, we do counterfactual exercises where we simulate how inflation would have evolved over the corresponding period had the economy not been hit by intermediate input-price shocks. For these experiments, we make use of intermediate input-price shocks we identified in the previous section. We run a counterfactual experiment by setting these shocks to zero and then compare the resulting inflation series to the actual one.

Before presenting our main analysis, it is helpful to study intermediate input-

Figure 3: **The Contribution of Input-Price Shocks to Inflation**



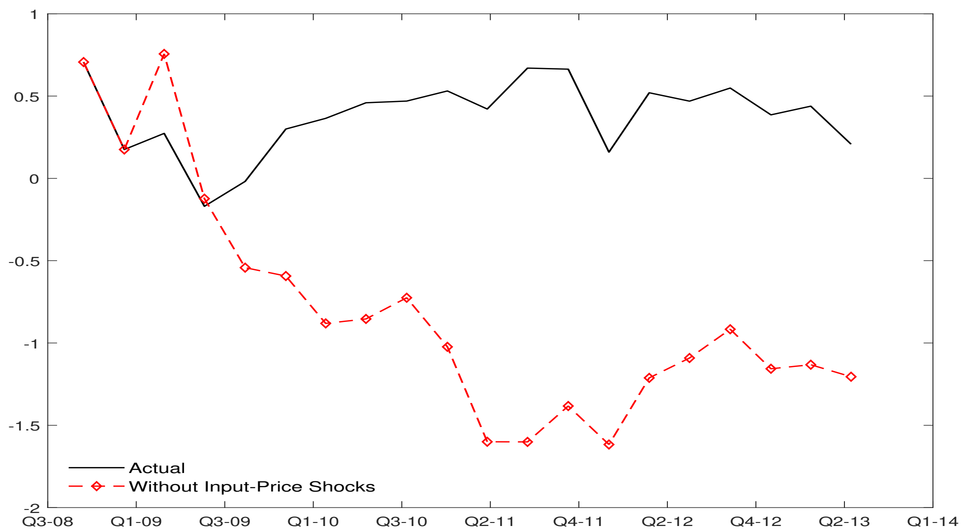
Note: *This figure plots the contribution of financial (red) and input-price (blue) shocks towards explaining fluctuation in actual inflation. The solid black line plots actual inflation.*

price shocks we identified. While Figure 2 displays these shocks, Figure 3 plots the contribution of such shocks towards explaining fluctuations in observed inflation.

As Figure 2 shows, apart from a brief period just before 2009, input-price shocks are positive before, during and after the Great Recession. Figure 3 indicates that these shocks had a significant inflationary effect on finished goods prices, offsetting the deflationary effect of the Great Recession. Figure 3 further shows that negative input-price shocks at the beginning of the crisis did not have a significant deflationary effect. This seems to be a result of the fact that these shocks were temporary.

We now turn to answer our main question: Had there been no change in intermediate input prices, what would have happened to inflation? Figure 4 provides

Figure 4: **Inflation Simulation without Input-Price Shocks**

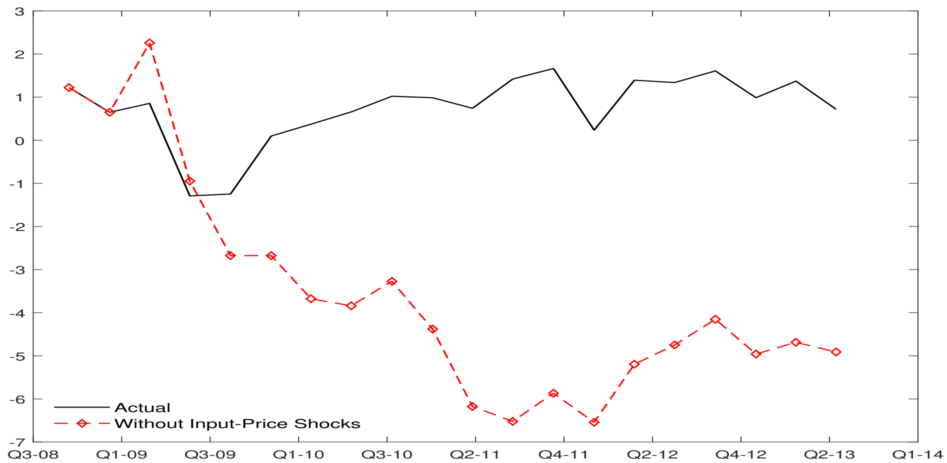


Note: *The solid black line is observed inflation. Dashed red line with diamonds is the simulation when there are no input-price shocks.*

an answer to this question by simulating our model after setting input-price shocks to zero from the first quarter of 2009 until the end of the simulation period (i.e. $\varphi_{2009Q1:2013Q2}$=0). When we compare the resulting simulated inflation series to actual inflation, we see that in absence of input-price shocks, there would have been persistent deflation. Inflation would have fallen more than it did and would have remained significantly negative for the whole period. Therefore, these findings suggest that the 'missing deflation' puzzle is a consequence of positive intermediate input-price shocks during the post-2008 period. There would have been no 'missing deflation' puzzle in absence of such shocks.

What is the intuition behind this result? Input-price shocks affect inflation

Figure 5: **Marginal Cost with and without Input-Price Shocks**



Note: *The solid black line is smoothed marginal costs,* $E[mc_t|Y_{1:T_{full}}]$*. The dashed red line is smoothed marginal costs without input-price shocks.*

through their effect on finished goods firms' marginal costs. In the model, inflation is determined by current and future marginal costs, which depends on intermediate input prices (see Equation 5). Marginal costs are much higher when input-price shocks are included. Figure 5 confirms this suggestion, plotting smoothed marginal costs (MC) both with and without input-price shocks. It appears that increasing intermediate prices almost completely offset the fall in marginal costs following the sharp contraction in economic activity. As a consequence, inflation did not fall much after the Great Recession.

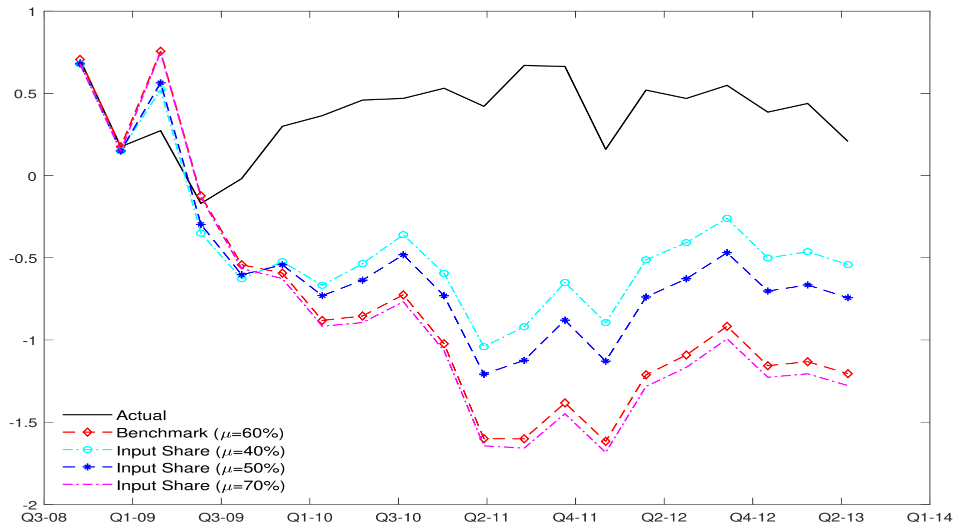
VI Robustness

In this section, we examine the robustness of our findings to alternative assumptions. We start by testing how sensitive results are to the share of intermediate inputs in finished goods production (μ). To this end, we re-estimate the model using alternative calibrations for the share of intermediate inputs and redo the simulations in section V. We consider three different calibrations: $\mu = 40\%$, $\mu = 50\%$ and $\mu = 70\%$. Figure 6 plots simulation results for these alternative calibrations as well as those from our benchmark case. The figure shows that our main conclusions that intermediate price shocks can help account for the missing deflation puzzle holds even when the intermediate input share is as low as 40%.

When estimating the model, we included long-run inflation expectations data as an observable to ensure that the model is consistent with the observation that inflation expectations remained anchored throughout the simulation period. We now quantify the role of inflation expectations to see if this assumption is essential for our results. To achieve this, first, we obtain model-implied inflation expectations by re-estimating our model after removing long-run inflation expectations data from estimation. We then construct a new dataset by replacing actual inflation expectations with model-implied inflation expectations. Next, using our model which was calibrated according to our benchmark parameter estimates and the new dataset, we extract shocks using Kalman Smoother. Finally, we use resulting shocks to do simulations.

Before presenting simulations results from this experiment, it is useful to compare model-implied inflation expectations with actual data. Figure 7 plots the two series

Figure 6: **Inflation Simulation without Input-Price Shocks**

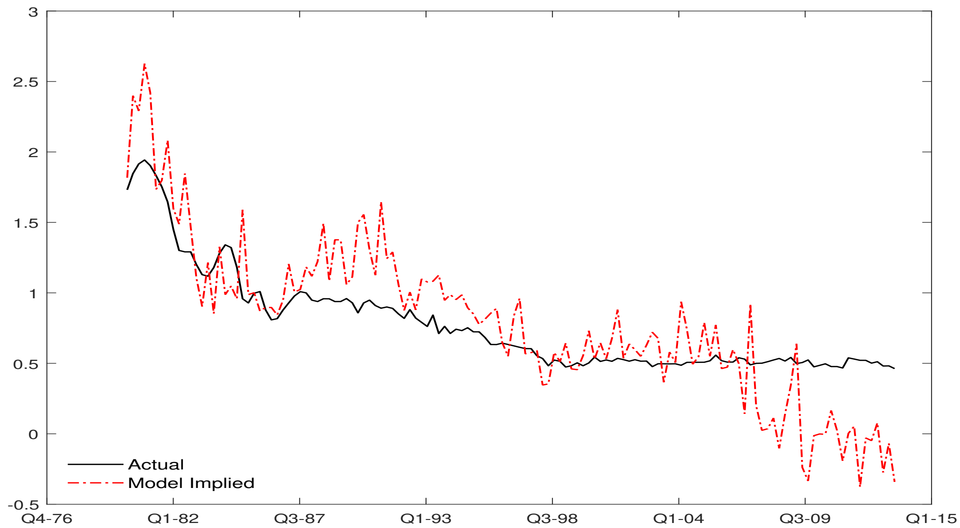


Note: *The solid black line is observed inflation. The dashed red line with diamonds shows the benchmark case when the intermediate input share is 60%. The other lines show simulation results for alternative calibrations of the intermediate input share.*

and shows that there are differences between them. During the simulation period, model-implied inflation expectations are lower than actual expectations. Model-implied expectations are also more volatile than actual series. Input shocks from both versions of the model are plotted in Figure 2. In the version of the model with model-implied inflation expectations, input shocks are slightly larger than those in the benchmark model with actual inflation expectations.

Figure 8 plots simulation results for inflation in the absence of input-price shocks (blue dash-dotted line). The figure also includes the corresponding inflation series from our benchmark model (red dash-diamond line). Results suggest that inflation would have fallen even more in absence of input-price shocks if inflation expectations

Figure 7: **Model Implied Inflation Expectations**



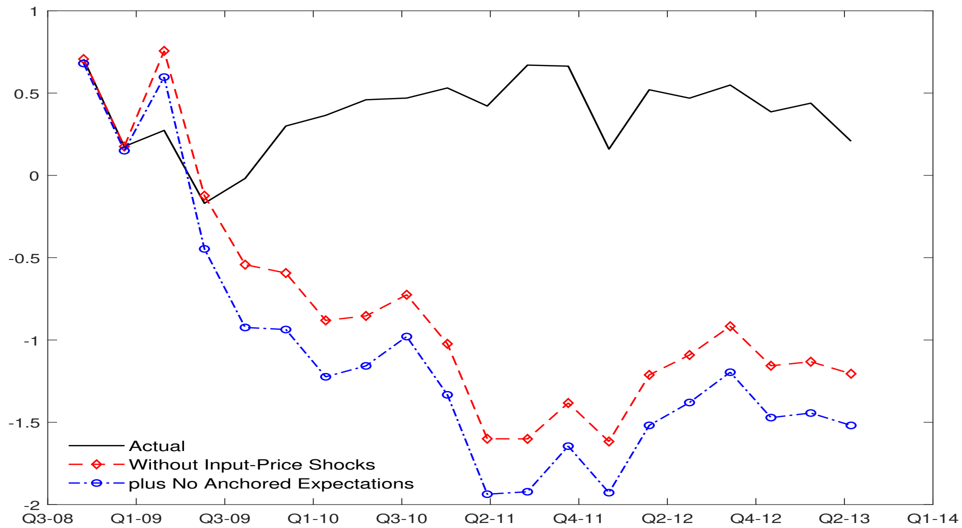
Note: *The solid black line is actual inflation expectations data used for baseline estimations in this paper. The dashed red line is (smoothed) inflation expectations obtained when the model is estimated without inflation expectations data.*

had not remained anchored. However, this difference appears to be small.

These findings show that while including inflation expectations as an observable helps to explain the puzzle, its contribution is small and accounting for the behaviour of intermediate input prices during the Great Recession is crucial in explaining the puzzle.

Finally, we estimate the model by extending our sample to 2018 and study the relationship between intermediate input prices and inflation beyond our simulation period which ends in the second quarter of 2013. Our main conclusion that intermediate input prices are an important determinant of inflation still holds.

Figure 8: **Inflation Simulation without Input-Price Shocks**



Note: *The solid black line is observed inflation. The red dashed-diamond line is the same as in figure 4. The blue dash-circle line is the simulation result for inflation when there are no input-price shocks and inflation expectations are unanchored.*

VII CONCLUSIONS

In this paper, we have reformulated the standard New Keynesian model to include the financial accelerator mechanism and to account for changes in intermediate input prices. In the new model, intermediate inputs are used as an additional factor input in the production of finished goods. We have estimated the model using quarterly US data. The estimated model is then used to do simulation exercises after the Great Recession to see if the reformulated model can account for the evolution of key macroeconomic variables over the simulation period.

We have shown that accounting for changes in intermediate input prices provides an explanation for the ‘missing deflation’ puzzle. Importantly, our model achieves this with an empirically relevant degree of price stickiness. In our model, despite persistent worsening of the output gap, inflation does not fall much during the post-2008 period because intermediate input prices were increasing during this time. Increases in intermediate input prices drove up firms’ marginal costs thus offsetting the deflationary effects of the Great Recession on inflation.

In this paper, we use a specific series to measure intermediate input prices. In practice, however, many variables can go into intermediate input costs. We choose this particular series, as it captures the main movements in intermediate input prices, which is sufficient for our purpose. However, ideally, a model aiming to capture intermediate input prices may need to make use of the information in the entire input-output matrix. In addition to this, sectors in the input-output matrix may be heterogeneous. Sectors that require more intermediate inputs may exhibit different inflationary responses. A model that accounts for such heterogeneity may generate new insights. We leave these issues as a matter for future research.

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